MIPP-NOTE-ANA-53

On MIPP Momentum Scale Based on RICH Rings

Andre Lebedev

February 7, 2005

1 Cherenkov angles for two different particles

Cherenkov half cone opening angle in case of gases is approximated by

$$\theta = \sqrt{2(1 - \frac{1}{n\beta})}. \tag{1}$$

Hence,

$$\frac{1}{n} = \beta \left(1 - \frac{\theta^2}{2}\right).$$

Then, if we have particles i and j with masses m_i and m_j radiating with Cherenkov angles θ_i and θ_j respectively,

$$\frac{\theta_i^2}{2} = 1 - \frac{\beta_j}{\beta_i}\left(1 - \frac{\theta_j^2}{2}\right) \approx 1 - \frac{\beta_j}{\beta i} + \frac{\theta_j^2}{2}, \tag{2}$$

where $\beta_{i,j}$ are speed of each particle divided by the speed of light.

Since $\beta = p/E$, from $E = sqrtp^2 + m^2$ in the relativistic limit we obtain

$$E = p\sqrt{1 + \frac{m^2}{p^2}} \approx p\left(1 + \frac{m^2}{2p^2}\right)$$

Therefore,

$$\beta \approx \frac{1}{1 + \frac{m^2}{2p^2}} \approx 1 - \frac{m^2}{2p^2} \tag{3}$$

Using equation 3, we then get

$$1 - \frac{\beta_j}{\beta_i} = 1 - \frac{1 - \frac{1}{2}m_j^2/p^2}{1 - \frac{1}{2}m_i^2/p^2} \approx \frac{1}{2}\frac{m_j^2 - m_i^2}{p^2}$$

Thus we obtain

$$\theta_i^2 - \theta_j^2 = \frac{m_j^2 - m_i^2}{p^2}, \tag{4}$$

which is independent of index of refraction.

2 Calculating momentum from RICH rings

If we assume that RICH ring fitter biases all ring radii in the same way (to be demonstrated through Monte Carlo simulation), then we can calculate momentum for pair of particles for each tune, since

$$p^2 = \frac{m_j^2 - m_i^2}{\theta_i^2 - \theta_j^2}. \tag{5}$$

.

Table 1 summarises the results for positive runs that were taken in December.

Tune	Ring radius (cm)				Calculated momentum (GeV/c)				
(GeV/c)	p	K	π	e	p-K	p-π	K-π	K-e	π-e
15	–	11.17	28.49	29.54	–	–	18.0	18.0	17.5
20	–	20.58	28.97	29.54	–	–	23.1	23.2	24.0
30	13.39	26.00	29.35	–	35.4	35.2	34.6	–	–
40	21.80	27.59	29.47	–	46.7	46.4	45.5	–	–
50	25.06	28.42	29.56	–	58.9	59.2	58.6	–	–
60	26.34	28.76	29.58	–	68.4	68.3	68.2	–	–

Table 1: Comparison of calculated momenta vs. nominal tune momentum.
Note: some of these runs were taken a few days apart, and therefore the density of CO2 in the radiator volume may be different, and direct comparsion of ring radii between two different momenta should not be made. That is why pion rings at 50 and 60 GeV/c tunes are larger than electron rings at 15 and 20 GeV/c tunes.